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Simple admittance expression derivation of an electrically dense loaded slot array at a material interface

Christos Mias, *Member, IEEE*, and Angelo Freni, *Senior Member, IEEE*

Abstract— The equivalent circuit admittance of a lumped element periodically loaded electrically dense array of slots at a material interface is derived for both, parallel and perpendicular, polarizations. The derivation is based on the methodology of Casey for deriving the equivalent circuit impedance of an unloaded wire mesh at a dielectric interface using the Wait-Hill formulation. Using the derived equivalent circuit admittance, a Booker type relation for unloaded wire grid and slot arrays, at general isotropic media interfaces, is obtained.

Index Terms— slot arrays, equivalent circuits, electromagnetic scattering by periodic structures

I. INTRODUCTION

STARTING from the Wait and Hill formulation [1], which is a method of moments (MoM) formulation, Casey proposed in [2] a methodology for deriving simple, closed-form, expressions for the equivalent circuit impedance of an electrically dense (period significantly smaller than the wavelength), dielectric-backed, unloaded wire mesh screen for both polarizations, parallel and perpendicular. An analogous derivation of simple, closed-form, expressions for the admittance of an electrically dense orthogonal array of slots (patches), at a material interface (which is also loaded with lumped elements, Fig. 1) to the best of the authors' knowledge, does not exist (and it is the subject of this paper). However, simple closed-form equivalent circuit impedance expressions for the unloaded array of slots on an air-dielectric interface were obtained, for both polarizations, in [3]. These expressions in [3] were obtained by combining the impedance of the wire grid on a dielectric substrate and a Booker type equation [4] in terms of the effective impedance (see (7) of [3] and [5],[6]). The wire grid impedance was obtained by a heuristic extension of Kontorovich's average boundary conditions (BC), (1)-(2) in [3], using the effective permittivity expression (3) of [3]. Reference [7] highlights limitations of the method used in [3] to obtain simple, closed-form patch array formulae. For example, the lack of a description or explanation of polarization decoupling and azimuthal independence of the obtained equivalent circuit impedance expression. Hence, [7] adapts Casey's assumption, (35) in [8], to the magnetic current and uses it as a starting approximation of their MoM derivation. Indeed, in [7], an analytical expression for the patch array equivalent circuit reactance is obtained, providing polarization and azimuthal angle description. However, [7] does not consider a material interface and lumped element loading. Furthermore, the derived analytical expression in [7] is not as simple as that of [3] since it contains an infinite sum, see (22) of [7]. Hence, the aim of the current paper is to derive a simple, closed-form, expression for the

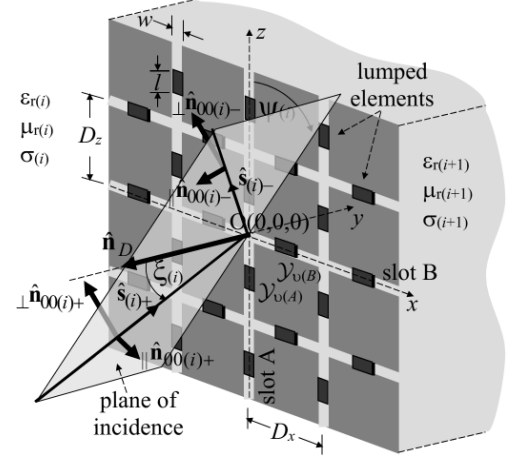


Fig. 1. Geometry of the loaded slot array.

admittance of an electrically dense, lumped element loaded, slot array at a general isotropic media interface (Fig. 1) by avoiding the limitations of the methodology in [3]: (i) the use of a heuristic extension; (ii) the use of a Booker type equation limited to dielectric materials (i.e. $\mu_{r(i)} = \mu_{r(i+1)} = 1$ in Fig. 1) and (iii) the angle of incidence limitation (see paragraph above (5) of [3]). Our derivation includes a novel Wait-Hill magnetic current formulation, shown in (10)-(11), for plane wave scattering from a lumped element loaded patch array at a general media interface (which is an extension of the free space magnetic current formulation in [9]) and subsequently applies the magnetic current approximation of [7]. Thus, our approach is related to the unloaded wire grid mesh approach and electric current approximation of Casey [2] and the free-space wire grid equivalent impedance derivation in [8] (with the loading applied as proposed in [10]) but here a loaded slot array is considered instead. In section III, using the derived equivalent circuit admittance, a Booker type relation for unloaded wire grid and slot arrays is obtained, that extends the Booker type relation for dielectric media, used in [3], to the case of general isotropic media. The benefits of deriving simple equivalent circuit expressions for electrically dense arrays are highlighted in [2],[3],[7],[11] (and papers citing them) and include physical insight and fast computation of the electromagnetic (EM) behavior of periodic structures in a variety of applications, such as EM shielding and absorption, frequency selective surface (FSS) radomes and artificial dielectrics. The more electrically dense is an array, the greater is the evanescence of the high order harmonics. This allows the use of simple transmission line models [3] to model single or multi-layer periodic structures [12].

II. DERIVATION OF THE WAIT-HILL FORMULATION

In this section a novel Wait-Hill formulation is derived for the solution of the problem depicted in Fig. 1. The worked-out plane wave transmission coefficient expression will be the starting point for calculating the desired analytical admittance expression for an

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electrically dense array, in Section III. The formulation requires a definition of the boundary condition (BC) - see (2) - at the two orthogonal slots of the reference unit cell (defined below). The BC (shown in [13] for a lossy dielectric window, and in [14] for an unloaded array) is in terms of the incident and scattered magnetic fields, the magnetic current, and the admittance per unit length as the slots are loaded with lumped elements (Fig. 1). The field notations used in [14] are employed here. The periodic part of the magnetic current is expressed as the sum of a periodic sawtooth function (to account for a sharp magnetic current discontinuity at the slot junction) with a harmonic function summation (9). By expressing the sawtooth function as a Fourier series, and substituting the resulting magnetic current expression into the BC ([1], [8] show the procedure), leads to simultaneous equations, (10) and (11). The use of the periodic sawtooth function and of the Kummer's transformation of a series in the simultaneous equations allows us to obtain the desired analytic admittance expression when the array period is electrically small. This is done in Section III where only three unknowns are retained of the Fourier series expression of the magnetic current. The geometry of the periodic structure under consideration is shown in Fig. 1. The media on either side of the array are assumed to be lossy with permittivity $\epsilon_{(v)} = \epsilon_0 \epsilon_r(v)$, permeability $\mu_{(v)} = \mu_0 \mu_r(v)$, conductivity $\sigma_{(v)}$, intrinsic impedance $\eta_{(v)}$ and wavenumber $k_{r(v)}$; where $v = i$ or $i+1$. The array coincides with the $y = 0$ plane. The incident magnetic field \mathbf{H}^{inc} , with amplitude H_0 , is assumed to be either parallel ($\mathbf{H} = \parallel$) or perpendicularly ($\mathbf{H} = \perp$) polarized and it is given by

$$\mathbf{H}^{inc}(\mathbf{R}) = H_0 e^{-jk_{r(i)}\hat{\mathbf{s}}_{(i)} \cdot \mathbf{R}} \mathbf{g}\hat{\mathbf{n}}_{00(i)} \quad (1)$$

where $\hat{\mathbf{s}}_{(i)\pm} = s_{x(i)}\hat{\mathbf{x}} \pm s_{y(i)}\hat{\mathbf{y}} + s_{z(i)}\hat{\mathbf{z}}$ is the direction vector and $\mathbf{R} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$. The polarization unit vector $\mathbf{g}\hat{\mathbf{n}}_{00(i)}$, shown in Fig. 1, is defined in (4.55)-(4.56) of [14]. In the reference unit cell ($|x| \leq D_x/2$, $|z| \leq D_z/2$) of Fig. 1, the slot axes coincide with the z -axis for reference slot A ($x_A = 0$) and with the x -axis for reference slot B ($z_B = 0$). The BC

$$[2\mathbf{g}\mathbf{H}^{inc} + \sum_{v=i}^{i+1} (-1)^{v \bmod i} (\mathbf{H}_{(v)A}^{sc} + \mathbf{H}_{(v)B}^{sc})] \cdot \hat{\mathbf{u}} = -Y_{L(K)}(u)M(u) \quad (2)$$

is imposed on each slot. In (2), which is an adaptation of the lossy dielectric window BC in [13], 'mod' is the modulo operator, i.e. $(v \bmod i)$ is zero for $v = i$ and one for $v = i+1$. In addition, $\hat{\mathbf{u}} = \hat{\mathbf{z}}$ for slot A and $\hat{\mathbf{u}} = \hat{\mathbf{x}}$ for slot B. The BC is imposed at \mathbf{R}_b by using, as in [14] and [15], the equivalence between the narrow slot width, w , and the 'wire' radius, b , i.e. $b = w/4$. Thus, for slot A, $\mathbf{R}_b(x, y, z) = \mathbf{R}(0, \pm b, z)$ and for slot B, $\mathbf{R}_b(x, y, z) = \mathbf{R}(x, \pm b, 0)$; the sign '-' corresponds to $v = i$ and the sign '+' corresponds to $v = i+1$. Over the reference unit cell of Fig. 1, the per unit length load admittance is $Y_{L(K)}(u) = \mathcal{Y}_{v(K)}/l$ for $(D_u - l)/2 \leq |u| \leq D_u/2$ and zero otherwise; where $K=A$, $u=z$ for slot A and $K=B$, $u=x$ for slot B. $\mathcal{Y}_{v(K)}$ is the lumped element admittance (see Fig. 1). Since $Y_{L(K)}(u)$ is periodic,

$$Y_{L(K)}(u) = \sum_{n=-\infty}^{\infty} Y_{n(K)} e^{-j2\pi n u / D_u} \quad (3)$$

with $Y_n(K) = (\mathcal{Y}_{v(K)}/D_u)(-1)^n \text{sinc}(n\pi l/D_u)$. The equivalent magnetic current along the reference slots A and B is given by ($x_A = z_B = 0$)

$$M_K(u) = M_{\Pi(K)} e^{-jk_{r(i)}s_{u(i)}u} = \sum_{p=-\infty}^{\infty} \mathbf{g}K_p e^{-j2\pi p u / D_u} e^{-jk_{r(i)}s_{u(i)}u} \quad (4)$$

$M_{\Pi(K)}$ is the periodic part of the magnetic current, $p=m$ for slot A and $p=q$ for slot B. In (2), based on the equivalence principle, the total scattered field in medium (v) for each slot contains the radiated field from each slot and its reflection from a perfectly electric conducting plane. Thus, $\mathbf{H}_{(v)K}^{sc} = \mathbf{H}_{(v)K\pm}^{(sc)} + \mathbf{H}_{(v)K\pm}^{ref(sc)}$; the signs '-' and '+'

correspond to $v = i$ and $v = i+1$, respectively. For slot A, the magnetic current is assumed [14] to be in the negative z -direction in medium $v = i$ and in the positive z -direction in medium $v = i+1$, hence

$$\mathbf{H}_{(v)A\pm}^{(sc)} = \frac{(-1)^{v \bmod i}}{\eta_{(v)} 2D_x} \sum_{m=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \mathbf{g}A_m \frac{e^{-jk_{r(v)}\hat{\mathbf{r}}_{mq(v)\pm} \cdot \mathbf{R}}}{r_{mq(v)}} \quad (5)$$

$$\times (\perp n_{mqz(v)} \perp \hat{\mathbf{n}}_{mq(v)\pm} + \parallel n_{mqz(v)} \parallel \hat{\mathbf{n}}_{mq(v)\pm})$$

The expression for $\mathbf{H}_{(v)A\pm}^{ref(sc)}$ is also given by (5). Similarly, for slot

B, $\mathbf{H}_{(v)B\pm}^{(sc)}$ and $\mathbf{H}_{(v)B\pm}^{ref(sc)}$ are given by (5) when the subscript x is substituted with z (and vice-versa) and $\mathbf{g}A_m$ with $\mathbf{g}B_q$. Subscripts on the right include the harmonic order, " m and/or q ", the Cartesian coordinate " x, y, z " for vector components, and the medium index which appears within round brackets. Polarization subscripts (\perp, \parallel) appear on the left of a variable. The direction vector of the m th harmonic is defined as $\hat{\mathbf{r}}_{mq(v)\pm} = r_{qx(v)}\hat{\mathbf{x}} \pm r_{mqy(v)}\hat{\mathbf{y}} + r_{mz(v)}\hat{\mathbf{z}}$ in medium v and its components can be obtained from (4.24), (5.3)-(5.5) of [14]. $\parallel, \perp n_{mqz(v)}$ is the z -component of the polarization unit vector $\parallel, \perp \hat{\mathbf{n}}_{mq(v)\pm}$ of the m th harmonic in medium v which is defined in (4.55)-(4.56) of [14]. The subscripts \pm on the right of variables correspond to the \pm sign of the direction vector $\hat{\mathbf{r}}_{mq(v)\pm}$ above. Starting from (2) and following the methodology of [8], which includes Kummer's transformation of a series to improve its convergence [1][8][10], leads to (1)-(2) of [9]. Similar to [9], the right hand side terms of (1)-(2) of [9] are given by $\Xi_A = -\mathbf{g}n_{00(i)z}2H_0$ and $\Xi_B = -\mathbf{g}n_{00(i)x}2H_0$ where it is assumed that $\exp(\pm jk_{r(i)}s_{y(i)}b) \approx 1$. However, in contrast with [9], the coefficients of the left hand side terms are not related to those of the wire grid by the Booker equation shown in [9] because of the material interface. Instead, they are given as follows (it is assumed that $1 + \exp(2jk_{r(v)}r_{mqy(v)}b) \approx 2$)

$$P_{q(B)}^{(m)} = -\frac{j\omega}{D_z} \sum_{v=i}^{i+1} \epsilon_{(v)} \frac{e^{-\Gamma_{mq(v)}b}}{\Gamma_{mq(v)}} \quad (6)$$

$$\times (\perp n_{mqx(v)} \perp n_{mqz(v)} + \parallel n_{mqx(v)} \parallel n_{mqz(v)})$$

$$\hat{Y}_{m(A)} = Y_{0(A)} + \sum_{v=i}^{i+1} \left\{ \frac{e^{-\Gamma_{m0(v)}b}}{\Gamma_{m0(v)}} (\perp n_{m0z(v)}^2 + \parallel n_{m0z(v)}^2) \right. \\ \left. - \frac{D_x}{\pi} \ln \left(1 - e^{-2\pi b/D_x} \right) [1 - r_{mz(v)}^2] + \Delta_{m(v)} \right\} \frac{j\omega \epsilon_{(v)}}{D_x} \quad (7)$$

with

$$\Delta_{m(v)} = \sum_{q=-\infty}^{\infty} \left\{ \frac{e^{-\Gamma_{mq(v)}b}}{\Gamma_{mq(v)}} (\perp n_{mqz(v)}^2 + \parallel n_{mqz(v)}^2) - \frac{e^{-\frac{2\pi q b}{D_x}}}{\frac{2\pi q}{D_x}} [1 - r_{mz(v)}^2] \right\} \quad (8)$$

where the propagation constant Γ_{mq} is defined in (17) of [8]. The expressions for $P_{m(A)}^{(q)}$, $\hat{Y}_{q(B)}$ and $\Delta_{q(v)}$ are given by (6), (7) and (8), respectively, when one replaces subscripts and superscripts $m0$, mz , $m(v)$, $|q|$, $m(A)$, $0(A)$, (m) , $q(B)$, (A) , (B) , z , x by $0q$, qx , $q(v)$, $|m|$, $q(B)$, $0(B)$, (q) , $m(A)$, (B) , (A) , x , z , respectively, and series index q by index m . As in [1], a discontinuous periodic sawtooth function f_Δ of

step Δ at the slot “junction” is used to improve the convergence of the results. As stated in [9], the implementation of f_Δ parallels that of [8]. Hence, starting from the f_Δ expression in (21) of [8], the periodic part of the magnetic current is expressed as

$$M_{\Pi(K)}(u) = \phi f_\Delta(u) + \sum_{p=-\infty}^{\infty} {}_9K'_p e^{-j2\pi pu/D_u} \quad (9)$$

where $\phi = 1$ for slot A and $\phi = -1$ for slot B . The magnetic current harmonic amplitudes ${}_9K_p$ are expressed in terms of the amplitudes ${}_9K'_p$ as shown in (23) of [8]. Substituting (9) in (1) and (2) of [9] leads to the following equation pair

$$\hat{Y}_{m(A)} {}_9A'_m + \sum_{n=-\infty}^{\infty} {}_9Y_{n(A)} {}_9A'_{m-n} - \sum_{q=-\infty}^{\infty} P_{q(B)}^{(m)} {}_9B'_q \quad (10)$$

$$+ U_m \Delta = \delta_{m0} \Xi_A$$

$$- \sum_{m=-\infty}^{\infty} P_{m(A)}^{(q)} {}_9A'_m + \hat{Y}_{q(B)} {}_9B'_q + \sum_{n=-\infty}^{\infty} {}_9Y_{n(B)} {}_9B'_{q-n} \quad (11)$$

$$- V_q \Delta = \delta_{q0} \Xi_B$$

where $\Xi_A = -\|, \perp n_{00z} 2H_0 \exp(-jk_r s_y b)$, $\Xi_B = -\|, \perp n_{00x} 2H_0 \exp(-jk_r s_y b)$, and δ_{m0} , δ_{q0} are Kronecker delta functions. U_m is given by

$$U_m = \hat{Y}_{m(A)} j \frac{(1-\delta_{m0})}{2\pi m} + \sum_{n=-\infty}^{\infty} {}_9Y_{n(A)} j \frac{(1-\delta_{(m-n)0})}{2\pi(m-n)} \quad (12)$$

$$+ \sum_{q=-\infty}^{\infty} P_{q(B)}^{(m)} j \frac{(1-\delta_{q0})}{2\pi q}$$

V_q is given by the right hand side of (12) if subscripts and superscripts $m0$, $m(A)$, (m) , $q(B)$, (A) , (B) , $(m-n)0$ are replaced by $0q$, $q(B)$, (q) , $m(A)$, (B) , (A) , $0(q-n)$, respectively, and series index q by index m . Denominator variables m and $(m-n)$ should be replaced by q and $(q-n)$.

As stated in [9], the extra equation needed for the unknown parameter Δ is given by (28) of [8] where the expressions for $G_{m(A)}$, $G_{q(B)}$ and W are also given. It is obtained from

$$\frac{1}{2} \left(\frac{\partial M_A}{\partial z} \Big|_{z=0^-} + \frac{\partial M_A}{\partial z} \Big|_{z=0^+} \right) = \frac{1}{2} \left(\frac{\partial M_B}{\partial x} \Big|_{x=0^-} + \frac{\partial M_B}{\partial x} \Big|_{x=0^+} \right) \quad (13)$$

which is the magnetic current dual version of (26) of [1].

III. EQUIVALENT CIRCUIT ADMITTANCE OF THE ELECTRICALLY DENSE ARRAY AND BOOKER TYPE RELATION

For the derivation of the equivalent circuit admittance of the slot array it is assumed that $D_x = D_z = D$. Furthermore, it is assumed that the loading of the slots is everywhere the same, i.e. $Y_{0(A)} = Y_{0(B)} = Y_0$. The approximation

$$M_{\Pi(K)}(u) \approx {}_9K_0 + \phi f_\Delta(u) \quad (14)$$

is made for an electrically small period. Hence, (10) (11) and (28) of [8] lead to the following reduced matrix equation,

$$\begin{bmatrix} \hat{Y}_{0(A)} & -P_{0(B)}^{(0)} & U_0 \\ -P_{0(A)}^{(0)} & \hat{Y}_{0(B)} & -V_0 \\ G_{0(A)} & -G_{0(B)} & W \end{bmatrix} \begin{bmatrix} {}_9A_0 \\ {}_9B_0 \\ \Delta \end{bmatrix} = \begin{bmatrix} -2H_0 {}_9n_{00z} \\ -2H_0 {}_9n_{00x} \\ 0 \end{bmatrix} \quad (15)$$

with $G_{0(A)} = jk_{r(v)} s_{z(v)}$, $G_{0(B)} = jk_{r(v)} s_{x(v)}$, $W = 2/D$; $v = i$ or $i+1$. In addition, if λ/D is sufficiently large so that $\Delta_{m(v)}$ in (8) and $\Delta_{q(v)}$ can be neglected, for $m=0$ and $q=0$, respectively, and for the approximations $s_x + \lambda q/D \approx \lambda q/D$ and $s_z + \lambda m/D \approx \lambda m/D$ to hold for $q \neq 0$ and $m \neq 0$, respectively, then the following expressions are

obtained for the terms in (15)

$$\hat{Y}_{0(A)} \approx Y_0 + \sum_{v=i}^{i+1} [(1-s_{z(v)}^2)X_{(v)} + (1-s_{z(v)}^2)\Theta_{(v)}] \quad (16)$$

$$\hat{Y}_{0(B)} \approx Y_0 + \sum_{v=i}^{i+1} [(1-s_{x(v)}^2)X_{(v)} + (1-s_{x(v)}^2)\Theta_{(v)}] \quad (17)$$

$$P_{0(A)}^{(0)} = P_{0(B)}^{(0)} \approx \sum_{v=i}^{i+1} s_{x(v)} s_{z(v)} \Theta_{(v)} \quad (18)$$

$$U_0 \approx \frac{j}{D} \sum_{v=i}^{i+1} \frac{s_{z(v)}}{k_{r(v)}} X_{(v)}, \quad V_0 \approx \frac{j}{D} \sum_{v=i}^{i+1} \frac{s_{x(v)}}{k_{r(v)}} X_{(v)} \quad (19)$$

where

$$X_{(v)} = -\frac{j\omega \epsilon_{(v)}}{\pi} (1 - e^{-2\pi b/D}), \quad \Theta_{(v)} = \frac{1}{\eta_{(v)} s_{y(v)} D} \quad (20)$$

The transmission coefficient expression is used to obtain the equivalent circuit admittance expressions. Since only the fundamental ($q = m = 0$) harmonic propagates in electrically dense arrays, the co-polarized and cross-polarized transmission coefficients are given by

$$T_{wg} = \frac{w E^{tran}(\mathbf{R}_{out})}{g E^{inc}(\mathbf{R}_{in})} = \frac{\eta_{(i)}}{\eta_{(i+1)}} \frac{w^* H^{tran}(\mathbf{R}_{out})}{g^* H^{inc}(\mathbf{R}_{in})} = -\frac{\eta_{(i)}}{\eta_{(i+1)}} \frac{F_{w^*g^*}}{H_0} \quad (21)$$

assuming $\mathbf{R}_{out} = \mathbf{R}_{in} = \mathbf{R}(x, 0, z)$. The symbols $g = \perp, \parallel$ ($g^* = \parallel, \perp$) and $w = \perp, \parallel$ ($w^* = \parallel, \perp$) represent the incident electric (magnetic) field polarization and scattered electric (magnetic) field polarization, respectively, and

$$F_{w^*g^*} = \left(\frac{g^* A_0 {}_w n_{00z(i+1)}}{\eta_{(i+1)} s_{y(i+1)} D_x} + \frac{g^* B_0 {}_w n_{00x(i+1)}}{\eta_{(i+1)} s_{y(i+1)} D_z} \right) \quad (22)$$

Solving (15) for A_0 and B_0 one can evaluate (22) and show that the cross-polarization is zero. Furthermore, the co-polarized transmission coefficient for perpendicular (electric field) polarization is,

$$T_{\perp \perp} = \frac{2(\eta_{(i)} / s_{y(i)})^{-1}}{\perp Y_s + (\eta_{(i)} / s_{y(i)})^{-1} + (\eta_{(i+1)} / s_{y(i+1)})^{-1}} \quad (23)$$

where $\eta_{(v)} s_{y(v)}$ is the transmission line impedance and $\perp Y_s$ is the equivalent circuit admittance of the loaded slot array

$$\perp Y_s = Y_0 D - \frac{j\omega D}{\pi} \ln(1 - e^{-2\pi b/D}) \sum_{v=i}^{i+1} \frac{\epsilon_{(v)}}{2} (1 + s_{y(v)}^2) \quad (24)$$

For parallel (electric field) polarization, the co-polarized transmission coefficient is

$$T_{\parallel \parallel} = \frac{2(\eta_{(i)} s_{y(i+1)})^{-1}}{\parallel Y_s + (\eta_{(i)} s_{y(i)})^{-1} + (\eta_{(i+1)} s_{y(i+1)})^{-1}} \quad (25)$$

where $\eta_{(v)} s_{y(v)}$ is the transmission line impedance and $\parallel Y_s$ is the equivalent circuit admittance of the array

$$\parallel Y_s = Y_0 D - \frac{j\omega D}{\pi} \ln(1 - e^{-2\pi b/D}) [\epsilon_{(i)} + \epsilon_{(i+1)}] \quad (26)$$

In [16], based on the Wait-Hill formulation and [2], the equivalent circuit impedance of a periodically loaded wire grid at a material interface was obtained for perpendicular (electric field) polarization ([16], Eq. 17)

$$\perp Z_g = Z_0 D - \frac{j\omega D}{\pi} \ln(1 - e^{-2\pi b/D}) \frac{\mu_{(i)} \mu_{(i+1)}}{\mu_{(i)} + \mu_{(i+1)}} \quad (27)$$

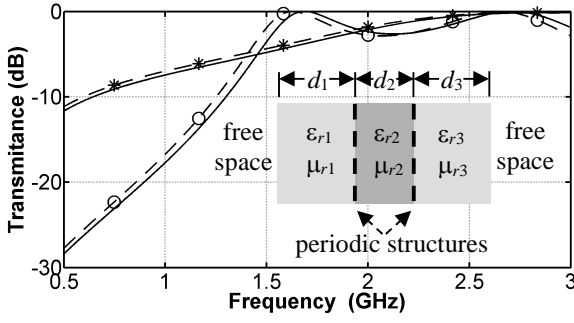


Fig. 2. Co-polarized transmittance results; $T_{\perp\perp}$: (○) and $T_{\parallel\parallel}$: (*). CST: dashed line. Transmission line analysis using (24) and (26): Solid line. The CST cross-polarization results are less than -30 dB.

and parallel (electric field) polarization ([16], Eq. 18)

$$\begin{aligned} \|Z_g = Z_0 D - \frac{j\omega\mu_{(i)}D}{\pi} \ln\left(1 - e^{-2\pi b/D}\right) \\ \times \left[\frac{\mu_{(i+1)}}{\mu_{(i)} + \mu_{(i+1)}} - \frac{1 - s_{y(i)}^2}{2} \frac{\epsilon_{(i)}}{\epsilon_{(i)} + \epsilon_{(i+1)}} \right] \end{aligned} \quad (28)$$

Ignoring the lumped elements and using Snell's law of refraction it can be readily shown that

$$\frac{\|Z_g}{\perp Y_s} = \frac{\perp Z_g}{\| Y_s} = \frac{\mu_{(i)}\mu_{(i+1)}}{[\epsilon_{(i)} + \epsilon_{(i+1)}][\mu_{(i)} + \mu_{(i+1)}]} \quad (29)$$

If media i and $i+1$ are (lossy) dielectrics ($\mu_{(i)} = \mu_{(i+1)} = \mu_0$) and the angle of incidence is in the x - y or z - y plane, then (29) leads to $(\eta_0)^2/(4\epsilon_{eff}/\epsilon_0)$ with $\epsilon_{eff} = [\epsilon_{(i)} + \epsilon_{(i+1)}]/2$ which are identical to (7) and (3) of [3], if medium i is free space. It is worth noting that (29) extends the Booker type relation for dielectric media, used in [3], to the case of general isotropic media.

IV. NUMERICAL EXAMPLE

An arbitrary multilayer configuration, shown in Fig. 2, is used to compare the transmission line results of (24) and (26) to those of CST [17]. The periodic structures (defined in Fig. 1) are identical. The periods are $D_x = D_z = D = 8$ mm, the slot width is $w = 4b = 0.4$ mm, the lumped element inductance value is $L = 20$ nH. The angles of incidence are $\xi = 70^\circ$ and $\psi = 22.5^\circ$. The material layer values are: $\epsilon_{r1} = \epsilon_{r3} = 1.5$, $\mu_{r1} = \mu_{r3} = 1$, $\epsilon_{r2} = 3$, $\mu_{r2} = 2$, $d_1 = d_3 = 20$ mm and $d_2 = 12$ mm. For the transmission line analysis, it is assumed that d_2 is sufficiently large such that evanescent harmonic coupling between the two periodic structures can be neglected. There is a good agreement between the full wave CST and the equivalent circuit transmission line analysis results (Fig. 2). An advantage of using the transmission line model based on the approximate expressions (24) and (26) is that its results are computed much faster than those of the CST software as no **linear system** construction and solution (as it is the case for the CST) is required.

V. CONCLUSION

The admittance of the electrically dense lumped element loaded orthogonal slot array was derived based on the methodology of Casey. The derivation avoids the limitations of a previous methodology based on a Booker type equation and Kontorovich's average boundary condition. In addition, a Booker type relation for unloaded wire grid and slot arrays, at general isotropic media interfaces, is obtained. As indicated in [15] the lumped elements may

be surface mount or in printed form. An example of loaded orthogonal slots is the slotted Jerusalem cross, where the region at the end caps may be considered as a printed form of a lumped element inductor. The developed analytic expression, see (24), indicates why the slotted Jerusalem cross is not an optimal frequency selective surface element (i.e. its admittance changes with frequency); an explanation is given in section 3.4 of [8] for a wire grid where a solution to the problem was suggested in the form of a lumped element inductor. **Hence**, for an orthogonal slot frequency selective surface, a lumped element capacitor in addition to the lumped element inductor is needed. Practical implementations of these lumped elements will be the subject of future work.

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